

A Unified Creep-Plasticity Model Suitable for Thermo-Mechanical Loading

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1.1 INTRODUCTION

Various unified creep-plasticity constitutive models have been proposed to account for time-dependent material behavior and temperature affects [1-6]. Material constants for the flow rule and the evolution equations are generally determined from a number of material behavior experiments. Model limitations may exist if material behavior predictions are considered outside the temperature or strain rate regime where the material constants have been established. New deformation mechanisms identified in deformation mechanism maps [7] could dramatically affect material behavior, but they may not be represented in the unified equations.

A constitutive model is needed that can account for relative rate insensitive material behavior obtained at low temperatures (plasticity deformation mechanism) as well as highly rate sensitive material behavior observed at high temperatures (power law creep and diffusional flow deformation mechanisms). This is necessary to provide an accurate time-dependent material model for a wide range of temperatures and strain rates.

A constitutive model with accurate rate and temperature predictive capabilities could then be checked with critical thermo-mechanical loading experiments. This is necessary to identify model capabilities and explore model limitations.

1.2 PURPOSE AND SCOPE

In this study:

- (1) a unified model is presented for isothermal and thermo-mechanical loading. Predictions are compared to experiments for a wide range of temperatures and strain rates.
- (2) deformation mechanisms operative for the alloys considered are incorporated into the constitutive equations.

2. THE CONSTITUTIVE EQUATIONS

The proposed unified creep-plasticity model contains two state variables. The state variable S_{ij}^c is the deviatoric back stress. It is a tensor that defines the center of the stress surface in deviatoric stress space. The second state variable K is the drag stress. It is a scalar that defines the radius of the stress surface in deviatoric stress space. In general, the state variables will evolve throughout the deformation history consistent with the Bailey-Orowan theory [8,9]. The coupled differential equations are

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^{in} + \delta_{ij} \dot{\epsilon}^{th} \quad (1)$$

$$\dot{\epsilon}_{ij}^{in} = f \left[\frac{(S_{ij} - S_{ij}^c)}{K} \right] \frac{(S_{ij} - S_{ij}^c)}{\sqrt{\frac{2}{3} (S_{ij} - S_{ij}^c)(S_{ij} - S_{ij}^c)}} \quad (2)$$

$$\dot{S}_{ij}^c = \frac{2}{3} h_\alpha \dot{\epsilon}_{ij}^{in} - r_\alpha S_{ij}^c \quad (3)$$

$$\dot{K} = h_K - r_K + \theta \dot{T} \quad (4)$$

where $\dot{\epsilon}_{ij}$ is the total strain rate, $\dot{\epsilon}_{ij}^e$ is the elastic strain rate (determined with Hook's law), $\dot{\epsilon}_{ij}^{in}$ is the inelastic strain rate, $\dot{\epsilon}^{th}$ is the thermal strain rate, δ_{ij} is the Kronecker delta, $S_{ij} = \sigma_{ij} - \sigma_{kk} \delta_{ij}/3$ is the deviatoric stress, $S_{ij}^C = \alpha_{ij} - \alpha_{kk} \delta_{ij}/3$ is the deviatoric back stress, σ_{ij} is the stress, α_{ij} is the back stress, \dot{S}_{ij}^C is the deviatoric back stress rate, K is the drag stress rate, \dot{T} is the temperature rate, h_α and h_K are the hardening functions, r_α and r_K are the recovery functions, f is the flow rule function, and θ is a temperature dependence term.

3. 1070 STEEL

A detailed systematic method has been established for determining the material constants from experimental data [10]. Two different flow rules shown in Fig. 1 are considered. Flow rule 1 is determined with plasticity deformation mechanism material behavior only. Flow rule 2 is determined from material behavior operating in the plasticity and power law creep deformation mechanism regimes. The flow rules and evolution equation constants for 1070 steel are shown in Table 1.

Flow rule 1 provides accurate material response simulations when the plasticity deformation mechanism is operative, but it is inaccurate for slow strain rate high temperature simulations when the power law creep deformation mechanism is operative. In Fig. 2, 600°C material response experiments and predictions with flow rule 1 are shown. For $\dot{\epsilon} = 2.0e-3 \text{ sec}^{-1}$ (plasticity), the predicted response is accurate. For $\dot{\epsilon} = 2.0e-6 \text{ sec}^{-1}$, a different mechanism is operative (power law creep), and the predicted stresses are significantly higher than the experimental values. Deformation mechanisms need to be incorporated into the flow rule and evolution equations to provide accurate strain rate effects.

Flow rule 2 accounts for the high strain rate sensitivity at higher temperatures. In Fig. 3, stable 600°C experimental and predicted response is shown. Accurate rate effects have been achieved. Through the flow rule and evolution equations the model can account for these two deformation mechanisms for a wide range of temperature and strain rates.

Thermo-mechanical total constraint material response is shown in Figs. 4 and 5. The mechanical strain is the sum of the elastic and inelastic strain components. Cycle numbers are shown on the figures at the maximum stress levels. These predictions are independent of the thermal experimental response and provide a good check for the constitutive model. For the thermo-mechanical response, plasticity (low temperature end) and power law creep (high temperature end) are both activated during a cycle. Predictions compare favorably with experiments.

4. CONCLUSIONS AND FUTURE WORK

An experimentally based unified creep-plasticity constitutive model has been implemented for 1070 steel. Accurate rate and temperature effects have been obtained for isothermal and thermo-mechanical loading by incorporating deformation mechanisms into the constitutive equations in a simple way.

Further work on low temperature material behavior for 1070 steel ($T < 400^{\circ}\text{C}$) and on different materials is presently being considered. Preliminary work on 304 stainless steel and nickel alloys show the model applicability to a variety of engineering materials.

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Table 1 1070 Steel Material

Constitutive Functions

$$\dot{\epsilon}^{th} = \dot{\beta T}$$

$$f\left[\frac{(S_{ij} - S_{ij}^c)}{K}\right] = A (\bar{\sigma}/K)^n$$

where $A = A' \exp [-\Delta H/RT]$

$$\bar{\sigma} = \sqrt{\frac{3}{2} (S_{ij} - S_{ij}^c) (S_{ij} - S_{ij}^c)}$$

$$h_{\alpha} = \begin{cases} a - b \bar{\alpha} & \text{for } S_{ij}^c \dot{\epsilon}_{ij}^{in} \geq 0 \\ a & \text{for } S_{ij}^c \dot{\epsilon}_{ij}^{in} < 0 \end{cases}$$

$$\bar{\alpha} = \left(\frac{3}{2} S_{ij}^c S_{ij}^c\right)^{1/2}$$

$$r_{\alpha} = c(\bar{\alpha}/\alpha^*)^d$$

where $c = c' \exp [-G/RT]$

$$h_K = 0, r_K = 0$$

$$E = e_1 - e_2 T$$

$$K_0 = h_1 - h_2 T$$

$$\theta = -h_2$$

Constitutive Constants

Flow Rule #1

$$n = 27.9$$

Flow Rule #2

$$\text{for } \bar{\sigma}/K \leq 1, n = 5.4$$

$$\text{for } \bar{\sigma}/K > 1, n = 27.9$$

$$\beta = 1.7 \times 10^{-5}/K$$

$$A' = 4.0 \times 10^9 \text{ sec}^{-1}$$

$$\Delta H = 210.6 \text{ KJ/mole}$$

$$a = 40000 \text{ MPa}$$

$$b = 100$$

$$\alpha^* = 100 \text{ MPa}$$

$$d = 3.2$$

$$c' = 5.0 \times 10^{14} \text{ sec}^{-1}$$

$$G = 248.4 \text{ KJ/mole}$$

$$\text{For } T \leq 713 \text{ K (440}^\circ\text{C)}$$

$$e_1 = 210710 \text{ MPa}, \quad e_2 = 31.0 \text{ MPa/K}$$

$$h_1 = 273.6 \text{ MPa}, \quad h_2 = 0.04 \text{ MPa/K}$$

$$\text{For } T > 713 \text{ K (440}^\circ\text{C)}$$

$$e_1 = 385260 \text{ MPa}, \quad e_2 = 275.7 \text{ MPa/K}$$

$$h_1 = 501.3 \text{ MPa}, \quad h_2 = 0.32 \text{ MPa/K}$$

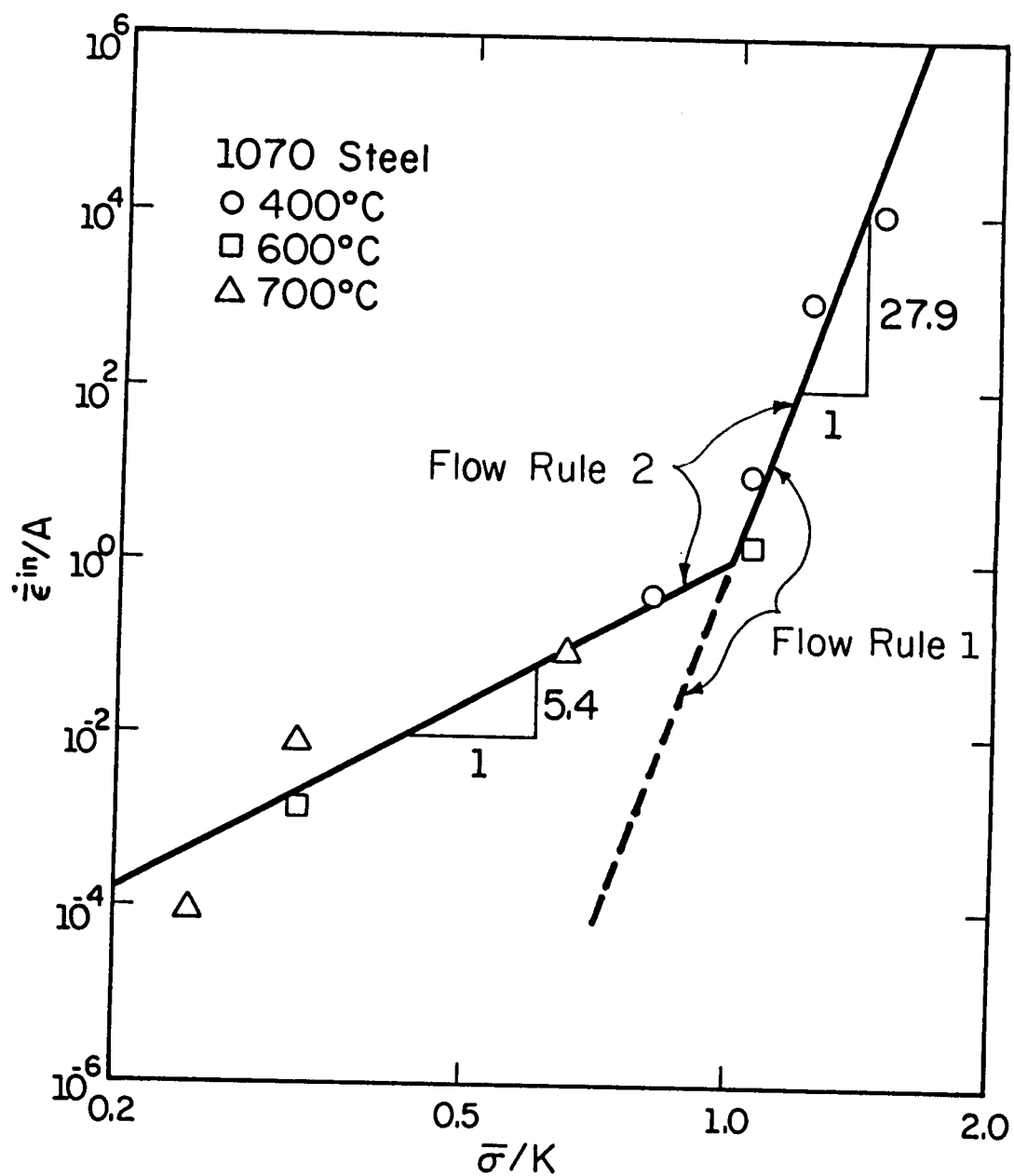


Figure 1 1070 Steel Flow Rule (400°C - 700°C)

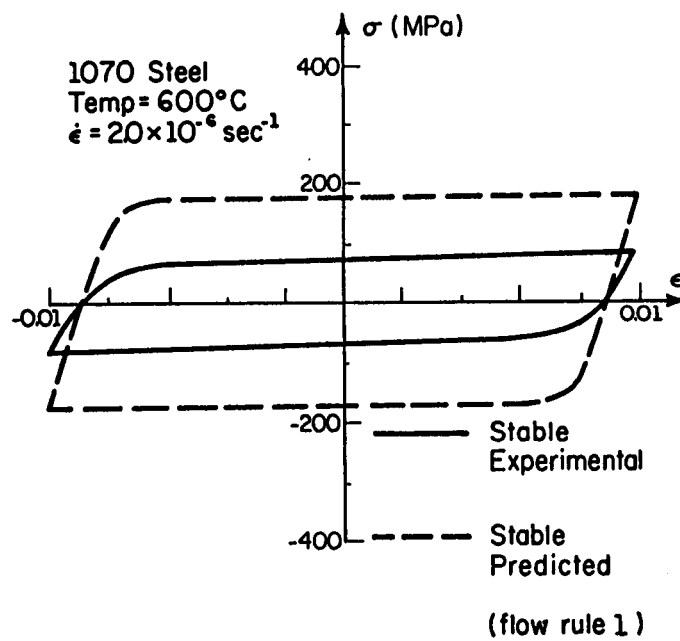
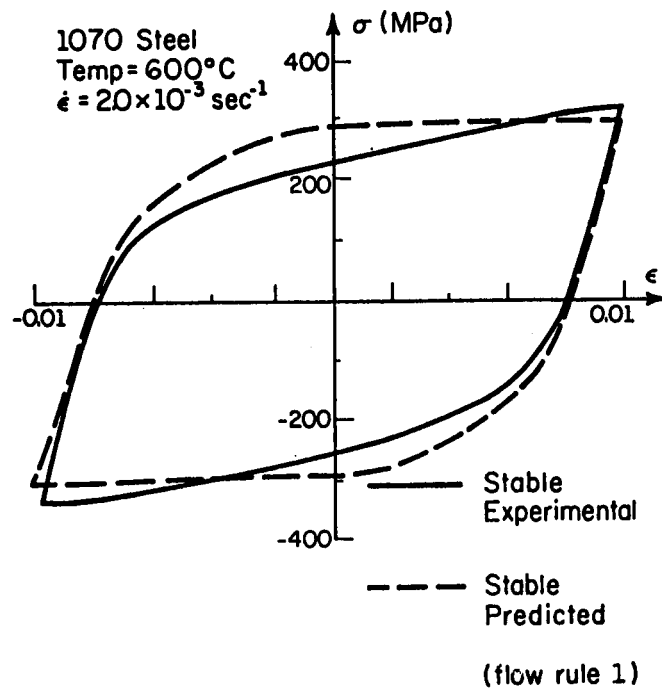


Figure 2 1070 Steel Experimental and Predicted Material Response, 600°C (flow rule 1)

(a) $\dot{\epsilon} = 2.0 \times 10^{-3} \text{ sec}^{-1}$ (b) $\dot{\epsilon} = 2.0 \times 10^{-6} \text{ sec}^{-1}$

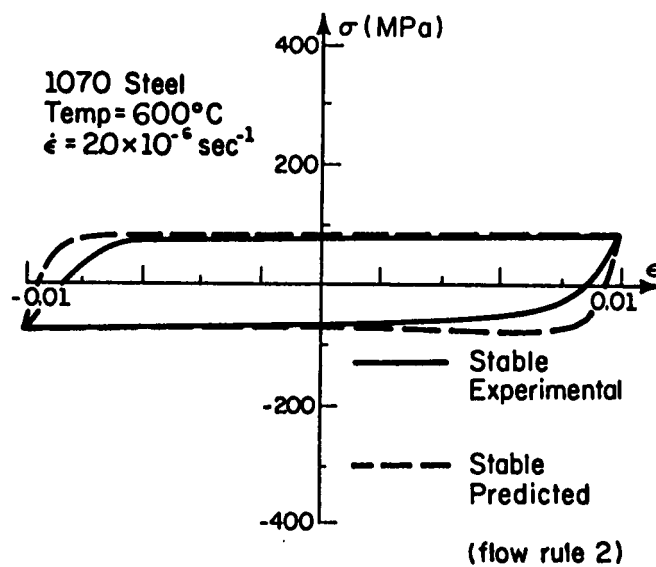
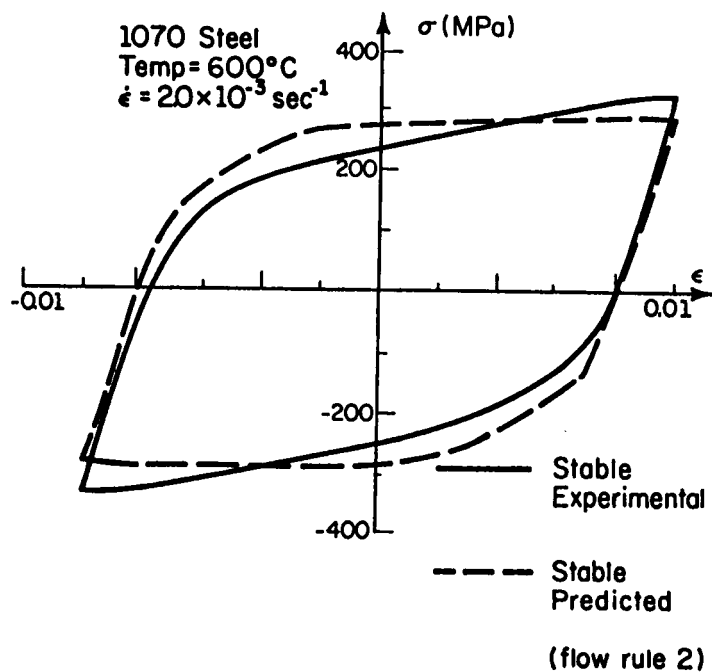


Figure 3 1070 Steel Experimental and Predicted Material Response, 600°C (flow rule 2)

(a) $\dot{\epsilon} = 2.0 \times 10^{-3} \text{ sec}^{-1}$ (b) $\dot{\epsilon} = 2.0 \times 10^{-6} \text{ sec}^{-1}$

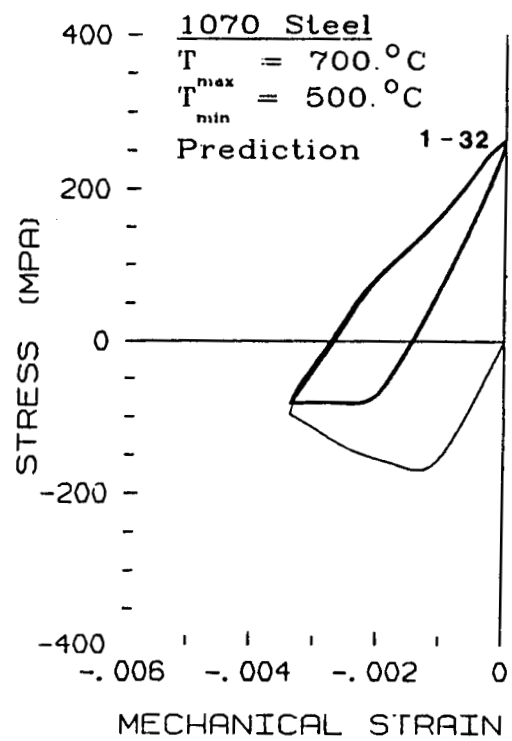
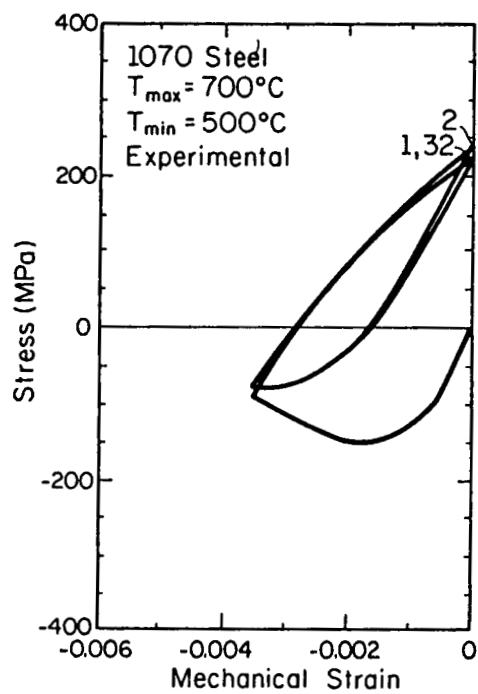


Figure 4 1070 Steel Thermo-Mechanical
 Material Response, 500°C - 700°C

(a) Experiment (b) Prediction

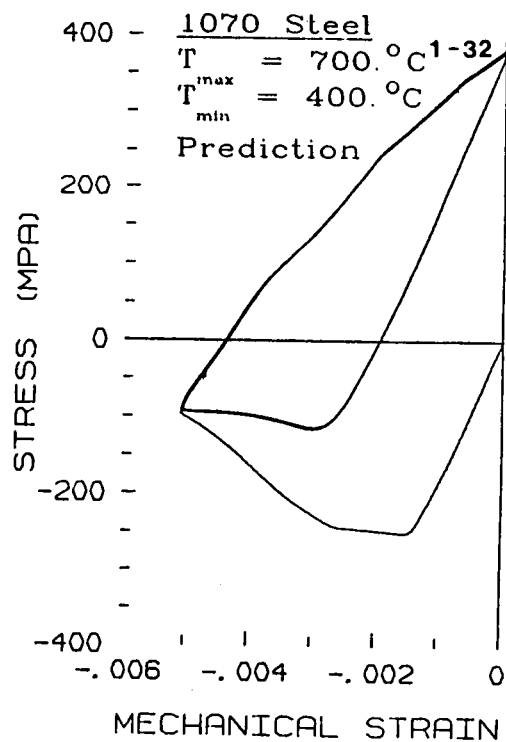
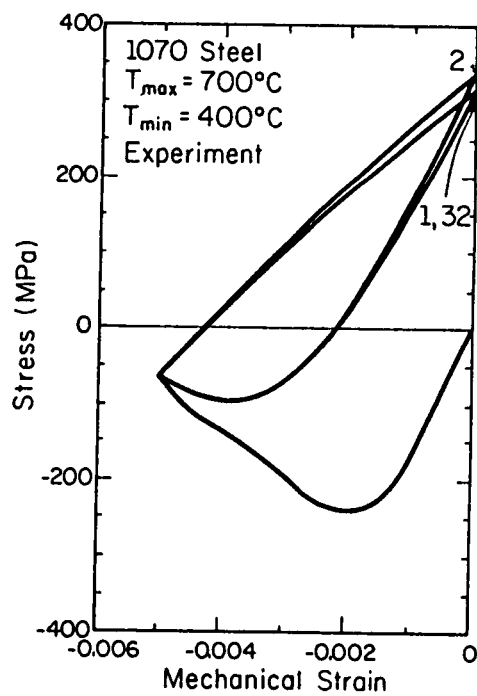


Figure 5 1070 Steel Thermo-Mechanical
 Material Response, $400^{\circ}\text{C} - 700^{\circ}\text{C}$

(a) Experiment (b) Prediction